Indian J. Phys. 71A (3), 343–347 (1997)

IJP A
— an international journal

# Phase transition in Quantum rotors in regular frustrations

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**Abstract**: The quntum rotor Hamiltonian, which is a $n$-component generalisation of the transverse Ising model has been studied in the spherical limit $n \to \infty$ with regular frustration in the interaction between the rotors. This frustration arises due to nearset neighbour ferromagnetic and next nearest neighbour antiferromagnetic interaction. The zero temperature phase diagram contains ferro phase, spirally modulated phase and a quantum disordered phase and the phase boundaries meet at a quantum Lifshitz point.

**PACS nos.**: 0.5.30.-d, 75.10.Jm

**Keywords**: quantum rotors, frustration, spherical limit.

The studies of phase transition driven by quantum fluctuations in pure and frustrated spin systems (e.g., classical Ising spin glass in the presence of transverse field) have been an exciting area of recent research [1]. The Ising model in a transverse field [1] and its $n$-component generaliastion, the rotor models ([2], [3]) are among the simplest systems to exhibit a zero-temperature quantum phase transition. In the absence of any quenched disorder the zero-temperature transitions in aforementioned models (in $d$-dimensions) have the same universality as the thermal phase transition in the equivalent classical model in $(d+1)$-dimensions. In the presence of quenched disorder, the situation is fairly complicated since the disorder is frozen in time, and thus has no dynamical fluctuation.

The quantum rotor Hamiltonian is written as [2]

$$H_r = -\frac{1}{2}\sum_{ij} J_{ij}\mathbf{x}_i\mathbf{x}_j + \frac{1}{2In}\sum_i L_i^2, \quad \text{with} \quad \mathbf{x}_i^2 = n, \tag{1}$$

where $\mathbf{x}_i$ are $n$-component, fixed length vectors occupying the $N$ sites of a $d$-dimensional square lattice and the operator $L_i^2 = 1/2\sum_{\mu\nu}(L_i^{\mu\nu})^2$ is the invariant formed from the asymmetric rotor angular momentum tensor $L_i^{\mu\nu} = x_i^\mu p_i^\nu - x_i^\nu p_i^\mu$. The moment of inertia term which determines the strength of the quantum fluctuations is denoted by $I$ and $J_{ij}$'s denote the interaction among the rotors. In the zero-temperature limit, when the quantum fluctuations are absent one encounters an ordered ground state where the rotational $O(n)$ symmetry is spontaneously broken. In the the opposite limit, $J_{ij} = 0$, the Hamiltonian $H_r$ possesses non-degenerate ground state and each site is a spherically symmetric $s$-wave state. With the increase of the quantum fluctuation term, namely $g = 1/I$, one eventually gets a quantum disordered (paramagnetic) ground state and hence a quantum phase transition from ordered to disordered phase. The $n = 1$ limit of the above Hamiltonian corresponds to classical Ising-like spin models in the presence of a transverse field [2].

We have studied the zero-temperature critical behaviour of the quantum rotor Hamiltonian in presence of regular frustration in the saddle point limit. The regular frustration in the Hamiltonian arises due to the competing nearest neighbour and next nearest neighbour interaction among the rotors. We essentially consider ANNNI model [4] like regular frustration where $J_{ij} = J_1$ when $i$ and $j$ are nearest neighbours and $J_{ij} = -J_2$ when $i, j$ are next nearest neighbours. If we define a measure of frustration given by $\alpha = |J_2|/J_1$, then the classical ground state ($g = 0$) is ferromagnetic for $\alpha < 1/4$ and spiral (nonzero ordering wave vector) for $\alpha > 1/4$ for $n \geq 2$ whereas for $n = 1$ for $\alpha > 1/2$ the ground state is spiral. We here consider a (2,1) rotor system in the saddle point limit, where (d,m) denotes that $m$ of the $d$ spatial dimension of the lattice incorporate frustration. The partition function the of the Hamiltonian (1) may be written as [3]

$$Z = \int D\mathbf{x} exp\left[\int_0^\beta d\tau \mathcal{L}(\mathbf{x}(\tau))\right], \tag{2}$$

with the effective classical action given by

$$\mathcal{L}(\mathbf{x}(\tau)) = -\frac{1}{2} I \sum_i |\partial_\tau \mathbf{x}_i(\tau)|^2 + \frac{1}{2}\sum_{ij} J_{ij}\mathbf{x}_i\mathbf{x}_j, \quad \text{with} \quad \mathbf{x}_i^2(\tau) = n, \tag{3}$$

where $\mathbf{x}_i(\tau)$'s are classical, imaginary time $(\tau)$-dependent $n$-component vectors of length $\sqrt{n}$. The saddle point free energy is given as

$$f_{sad} = -\ln \int Dx_{\mathbf{q}},(i\omega_n) Dx_{-\mathbf{q}},(-i\omega_n)$$

$$\exp\left[-\frac{I}{2}\sum_{n,\mathbf{q}}(\omega_q^2 + \omega_n^2) x_{\mathbf{q}},(i\omega_n) x_{-\mathbf{q}},(-i\omega_n)\right], \tag{4}$$

where $\omega_n$ are the Matsubara frequencies and $\omega_q^2/g = \lambda - J(q)$, $\lambda$ being the (global) undetermined multiplier to account for the constraint $\sum \mathbf{x}_i{}^2 = n$. The saddle point condition is given as

$$\sum_q \frac{g}{2\omega_q} coth(2\beta\omega_q) = 1$$

$$\sum_q \frac{g}{2\omega_q} = 1 \quad \text{for} \quad T = 0. \tag{5}$$

Following Berlin and Kac [5] the quantum critical condition is obtained as

$$\sum_q \frac{\sqrt{g_{c0}}}{(\lambda_s - J(q))^{1/2}} = 1, \tag{6}$$

where $\lambda_s$ is the maxima of $J(q)$. Defining $p = 1 - 4\alpha$ one finds for a $(d, 1)$ system

$$J(q) = \cos q_1 + \ \dots\ + \cos q_d - 1/4(1-p)\cos 2q_1,$$

so that [6]

$$\lambda_s(p) = 1/4(4d - 1 + p), \quad p > 0 \quad (\text{Ferro} - \text{Para transition})$$

$$\lambda_s(p) = 1/4(1-p)^{-1}[(4d-1)-(4d-2)p+p^2), \ p < 0 \ (\text{Spiral} - \text{Para transition})$$

The saddle point phase diagram in the $(g - p)$ plane contains ferromagnetic, spiral and quantum disordered phase. The phase boundary separating the ferro to para phase and ferro to spiral phases meet at a (2,1) quantum Lifshitz point ($p = 0$). Like the classical case [6], in the zero-temp quantum phase transition of rotors we define the quantum Lifshitz point as a special point in the phase boundary where ferro-para and helical-para phase meet. The details of the calculation and the phase diagram will be be published elsewhere [7]. The exponents for zero-temperature transition in the rotors in the spherical limit ($n \to \infty$) are the exponents associated with the thermal phase transition in the spherical model with one added dimension. For $(2, 1)$ system the exponents $\nu$ and $\eta$ come out to be 0 and 1 respectively.

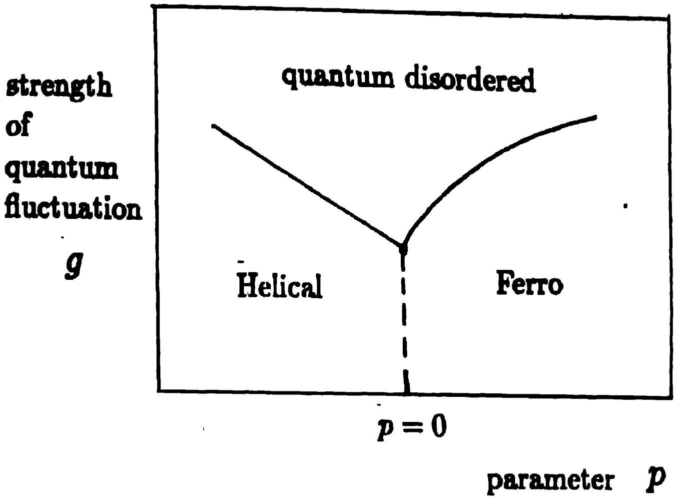


Fig. 1.1 The schematic phase diagram of the quantum rotor Hamiltonian in the prese nce of regular frustration.

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